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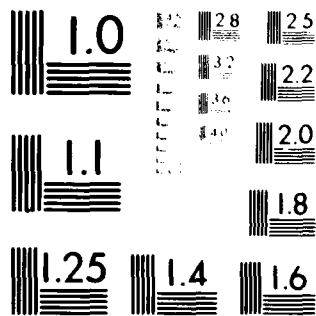
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ACOUSTIC AND VIBRATION FIELDS GENERATED BY RIBS
ON A FLUID-LOADED PANEL.

I. PLANE-WAVE PROBLEMS FOR A SINGLE RIB.

by

D.G. Crighton and G. Maidanik

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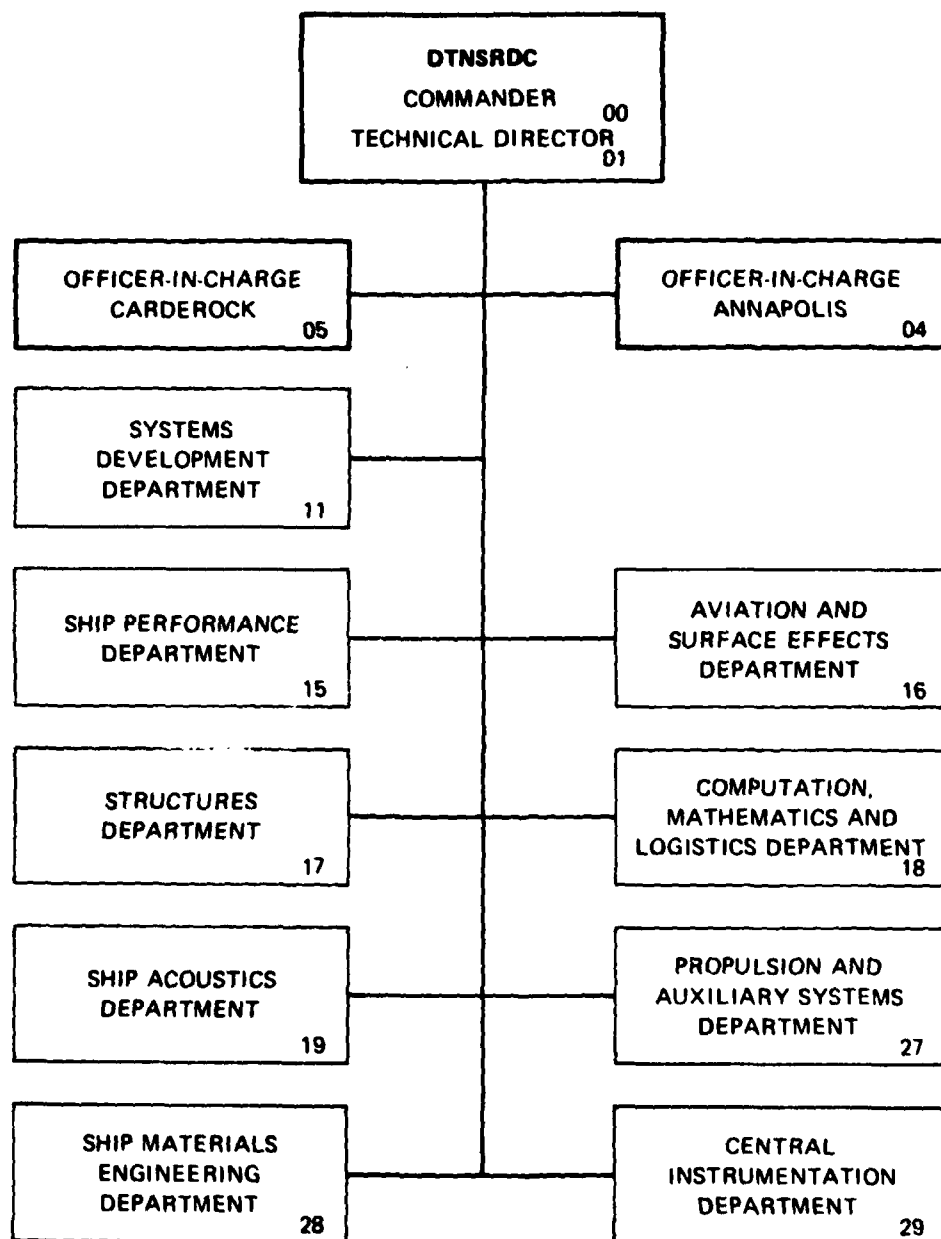
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ACOUSTIC AND VIBRATION FIELDS GENERATED BY RIBS ON A FLUID-LOADED PANEL
I. PLANE-WAVE PROBLEMS FOR A SINGLE RIB

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ABSTRACT

This paper presents an analytical study of the interaction between incident wave fields and a single rib on a fluid-loaded panel. The panel is modeled as an infinite membrane (with frequency dependent tension to partially simulate the dispersion characteristics of a thin elastic plate), and the incident waves are taken as plane structural or acoustic waves at normal and oblique incidence on the rib. Our principal concern is with the structural wave field transmitted across the rib (in the case of infinite mechanical impedance and finite nonlocal rib impedance) though the nonspecular acoustic field scattered by the panel-plus-rib is also examined. Matched asymptotic expansions are used to construct analytical approximations covering the entire frequency range, utilizing the smallness of a "fluid-loading-at-coincidence" parameter. The analytical results recover numerical results obtained elsewhere in appropriate regions of parameter space, complement them by providing simple approximations in those regions (typically of rapid variation or of heavy fluid loading) where numerical methods are difficult to implement, and reveal the physical aspects of fluid loading in effecting energy transfer across ribbed structures.

ADMINISTRATIVE INFORMATION

This work was performed under DTNSRDC Contract Nos. N68171-78-M-9493 and N68171-79-M-8380, and DTNSRDC Job Order 1-1902-005-01. Dr. Crighton is Head of the Department of Applied Mathematical Studies, University of Leeds, Leeds, England.

1. INTRODUCTION

This report attempts to give a comprehensive analytical description of acoustic and vibration phenomena associated with the interaction between plane acoustic or structural waves and a single mechanical line constraint on a fluid-loaded surface. Specifically, the phenomena analyzed concern the transmission of free subsonic surface wave energy

across the constraint (the "rib"), the generation of an acoustic field in the transmission process (i.e., the scattering of vibration energy into sound), and the non-specular field generated when a plane propagating acoustic wave is incident upon, and reflected by, the surface. Studies of these processes have, of course, been made before; for the transmission coefficient see [1,2], while the scattered acoustic field has recently been examined in [3] and [4].

None of these studies has, however, been able to cover the whole frequency range of interest because they have not exploited the fact that an appropriately defined fluid loading parameter ϵ is invariably small. This smallness can cause difficulties in numerical calculations, as happened for example, in [2] where it was not possible to evaluate the free wave energy transmission at low frequencies and at frequencies "close to coincidence." An explicit recognition of the smallness of ϵ can, nonetheless, be turned to advantage with the aid of modern perturbation theory. This can be used to construct a sequence of analytical approximations describing rib-associated phenomena, and to ensure that the approximations blend smoothly into one another so as to cover the entire range of frequencies.

A first step in this direction was taken in [5] where approximations for small ϵ , covering the entire frequency range, were derived for the line transfer admittance A_{∞} and the line drive admittance A_0 of a fluid-loaded surface. The surface considered was a membrane with frequency-dependent tension to simulate the dispersive wave characteristics of a thin elastic plate. Naturally, of course, such a device fails to reproduce the near-field behavior associated with shear rather than tension, but

it was still felt that the membrane model could give useful results—in particular, in the form of physical insight—for the role played by fluid loading in the interaction between structural waves and surface discontinuities. This role was discussed in [2]; the essential point is that fluid loading is able to "mend" a surface discontinuity in a way that in some frequency regimes is more or less complete, so that a structural wave can experience total transmission across the rib—even if the rib has infinite mechanical impedance (so that total reflection would occur in the absence of fluid loading).

In the light of these remarks it is then appropriate to consider whether the transmission might not be reduced in the presence of fluid loading if a suitable finite mechanical impedance of the rib were utilized. This will be considered in detail in Section 4 below. It is also appropriate to inquire whether studies of structural waves incident normally on a rib adequately cover applications in which free structural waves are incident on a rib from all possible directions. The problem of oblique incidence can be readily related to that for normal incidence, as explained in Section 5 below, where the requisite transformations are given. It turns out, however, that when the transformations are applied to the asymptotic expressions, the transformed expressions are not uniformly valid for certain particular directions of incidence. Separate studies have to be made for those directions, and the results are briefly reported in Section 5—briefly, because it turns out that no new phenomena arise in the oblique incidence case. In particular, in the frequency range for which the "mending mechanism" of fluid loading is inefficient for normally incident waves, it remains

comparably inefficient for all obliquely incident waves. Thus, there appears to be no significant filtering (i.e., selective transmission) of waves incident from preferred directions, at least when the constraining rib has infinite mechanical impedance.

One is then again prompted to consider the effect of finite rib impedance, and in the case of oblique incidence a local reaction impedance is inappropriate. The rib is therefore represented as a beam (or rather, as the plate has been replaced by a membrane, by a string) with its own nonlocal reaction. At each frequency ω , a certain direction of incidence corresponds to resonance of the rib, the free wavenumber of the rib being matched by the incident wavenumber parallel to the rib, and it is shown that there is perfect transmission of such oblique waves regardless of fluid loading effects. In particular, there appears to be no way of using fluid loading effects to significantly inhibit this selective transmission of oblique waves which cause the rib to resonate.

In all this, the membrane model of the surface will be retained. Extension to the case of a thin elastic plate may be a useful development in due course, but a more urgent need at present is to extend these studies of plane wave incidence on a single rib to cases where the incident energy is generated by, say, a force at a finite (small or large) distance from a rib, and where there are several regularly or irregularly spaced ribs. These extensions will form Part II of this report. A completely general formulation covering all these aspects (arbitrary numbers and locations of ribs, arbitrary driving excitation, arbitrary (homogeneous) surface response operators, etc.) is being given under separate cover [6];

the point of Parts I and 2 of the present work is to give a quantitative assessment, in simple model configurations, of the various coupling mechanisms implicit in the general formulation.

In the following sections, a surface or panel of the membrane type occupies the plane $y = 0$, with fluid of density ρ_0 , and sound speed c_0 , in $y > 0$, and a vacuum in $y < 0$ (with trivial modifications to allow for the presence of identical fluid in $y < 0$). A single "rib," whose action on the panel may be represented at each frequency ω by a scalar admittance, lies along the line $y = 0$, $x = 0$, and cases of normal incidence involve waves propagating in the x -direction only. For oblique incidence, the rib admittance is allowed to depend upon the wavenumber k in the z -direction, as well as on the frequency. The time factor is taken as $\exp(-i\omega t)$, $\omega > 0$, and is omitted. Extensive use will be made of methods and specific results reported in [5]; the readers familiarity with [5] will be assumed.

2. FREE WAVE TRANSMISSION: NORMAL INCIDENCE

A subsonic free surface wave with velocity $V_0 \exp(ik_1 x)$ is incident from $x = -\infty$ on a single rib at $x = 0$. Write the total velocity of the panel as

$$v_{\text{total}}(x) = V_0 \exp(ik_1 x) + v_r(x)$$

and define the rib admittance A_r so that the force into the fluid with which the rib acts on the panel is given by

$$v_{\text{total}}(x = 0) = A_r F.$$

The single scalar A_r completely defines the effect of a line mechanical constraint on a membrane type of surface. Next, relate $v_{\text{total}}(0)$ to the amplitude V_o of the incident wave through the introduction of the line drive admittance A_o , giving (by definition)

$$v_r(0) = A_o F$$

so that

$$F = \frac{V_o}{A_r - A_o}$$

This rib-induced force then itself produces a subsonic surface wave field transmitted to $x = \pm\infty$, and given by

$$v_r(|x| \rightarrow \infty) \sim A_\infty F \exp(ik_1 |x|)$$

A_∞ being the line transfer admittance to large distances. The total surface wave field at infinity to the right of the rib is thus

$$v_{\text{total}}(x \rightarrow +\infty) \sim \left(1 + \frac{A_\infty}{A_r - A_o}\right) V_o \exp(ik_1 x)$$

so that a transmission coefficient may be defined as

$$T = 1 + \frac{A_\infty}{A_r - A_o} \quad (2.1)$$

which is precisely Eq. (16) of [2] in a different notation.

Now asymptotic approximations have been given in [5] for the admittances A_∞ and A_0 , utilizing the fact that an appropriately defined fluid-loading parameter is small in most applications. Clearly, these approximations can be used to evaluate T for any prescribed value of the complex rib admittance A_r . Here, in order to emphasize the role played by fluid-loading, we evaluate the transmission for a rib of infinite mechanical impedance ($A_r = 0$), the transmission then being non-zero only by virtue of the fluid coupling. We use the dimensionless parameters of [5], namely

$$\Omega = \omega/\omega_g, \quad \epsilon = \rho_o c_o / m \omega_g,$$

the specific mass and coincidence frequency, m and ω_g , respectively, being assignable quantities. The transmission coefficient is then

$$T = 1 - \frac{2\epsilon}{\Omega^{\frac{1}{2}}} \frac{B_\infty}{B_0}, \quad (2.2)$$

where the dimensionless admittances B_∞ , B_0 were evaluated in [5]. The transmission coefficient will now be evaluated for various regimes in the dimensionless frequency domain Ω .

If $\epsilon \ll 1$ and $0 < \Omega < 1$ (but Ω not close to 0 or 1) we find

$$T_\Omega = \frac{\epsilon(1+D)}{2\Omega^{\frac{1}{2}} D^{3/2}} \left\{ 1 + \frac{2i}{\pi} \left(\frac{D^{\frac{1}{2}}}{1+D} \right) - \frac{2i}{\pi} \ln \left(\frac{1+D^{\frac{1}{2}}}{\Omega^{\frac{1}{2}}} \right) \right\} + O(\epsilon^2) \quad (2.3)$$

where $D = 1 - \Omega$. The $O(\epsilon^2)$ term can also be calculated from results given in [5]. A numerical evaluation of $|T_\Omega/\epsilon|$ is given in Table 1, and

TABLE I - VALUES OF $|T_{\Omega}/\epsilon|$ CALCULATED FROM EQUATION (2.3)

Ω	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$ T_{\Omega}/\epsilon $	4.59	3.28	2.91	2.88	3.10	3.63	4.72	7.52	18.34

the results agree with the numerical plot given in [2, Figure 2] by Maidanik, et al, except when Ω is small or close to unity. The existence of a minimum value of $|T_{\Omega}/\epsilon|$ close to $\Omega = 0.4$ should be noted. Note also that Eq. (2.3) predicts infinite transmission as $\Omega \rightarrow 0$, with

$$T_{\Omega} \sim \frac{\epsilon}{\Omega^{3/2}} \left(1 + \frac{i}{\pi} - \frac{2i}{\pi} \ln \frac{2}{\Omega^{1/2}} \right)$$

and infinite transmission as $\Omega \rightarrow 1 -$, with

$$T_{\Omega} \sim \frac{\epsilon}{2(1-\Omega)^{3/2}}$$

These predictions are unacceptable and clearly indicate that the approximations leading to Eq. (2.3) are not uniformly valid. We find, in fact, that complementary "matching" expansions are needed when $|1-\Omega| = O(\epsilon^{2/3})$ and when $\Omega = O(\epsilon)$.

Note finally that Eq. (2.3) implies that only a very small fraction, $O(\epsilon)$, of the incident energy is transmitted when $0 < \Omega < 1$.

We turn next to $\Omega > 1$ (but Ω not close to 1) and find that

$$T_{\Omega} = 1 - \left(\frac{2\epsilon^2}{\Omega^{3/2}(\Omega-1)^3} \right) + O(\epsilon^3). \quad (2.4)$$

Again, the $O(\epsilon^2)$ term can be obtained from [5], but serves only to show that Eq. (2.4) remains valid for arbitrarily large values of Ω . Thus the transmission is virtually total at all frequencies above coincidence, a surprising result at first sight, and one not in agreement with the numerical evaluation given in Figure 2 of Maidanik et al [2]. Substantial transmission was found there^{*} only in the immediate vicinity of $\Omega = 1$, the transmission decreasing rapidly as Ω increases beyond 1. Recall, however, that the results in Eqs. (2.3) and (2.4) apply only to the transmission of the subsonic surface wave whether $\Omega < 1$ or $\Omega > 1$. When $\Omega > 1$, the wavenumber k_1 is only slightly greater (by $O(\epsilon^2)$) than the acoustic wavenumber k_0 , and therefore the acoustic impedance $\rho_0 \omega / (k_1^2 - k_0^2)^{1/2}$ is very high. This is reflected in a very low value of A_∞ when $\Omega > 1$, whereas the drive admittance does not change so dramatically as Ω increases through unity; and the effect of the rib, represented in T by A_∞/A_0 , is therefore small when $\Omega > 1$. In other words, because the wavenumber k_1 is only marginally subsonic, the rib can have little effect on it, and fluid loading completely "mends" (cf. [2]) the surface discontinuity presented by the rib.

Now in the calculation of Maidanik et al [2], a Fourier integral for the transmission at a point $x_1 > 0$ of the total field generated by a prescribed force at $x_0 < 0$ was evaluated numerically for large, but finite, values of x_0 and x_1 , no attempt being made to separate the field out

^{*}Private communication from G. Maidanik; in the published paper the curve for $\Omega > 1$ was inadvertently omitted.

into its various components and to examine the separate transmission of each of these. When $\Omega < 1$, the dominant mode generated and transmitted is, of course, the subsonic free mode so that our results and those of [2] would be expected to agree. When $\Omega < 1$, however, the dominant wave in the surface, at any rate for moderate distances x_0, x_1 , is the "leaky wave field" (see [7]), rather than the subsonic wave. Ultimately, all the leaky wave energy is lost to the radiation field, and we are left (as $x_0, x_1 \rightarrow \infty$) with just the subsonic wave, but the distances must satisfy $k_m x \gg \epsilon^{-1}$ for this to be the case. One is led to believe, therefore, that the calculations of Maidanik et al actually refer to the transmission of the leaky wave across the rib. To substantiate this, use the expression (4.3) of [5] for the transfer admittance to large ($k_m x \gg 1$) but not too large ($k_m x \gg \epsilon^{-1}$) distances of the leaky wave, and in place of (2.4) one finds

$$T(\text{leaky}) = \epsilon e^{-\pi i/2} \left[\frac{2 - \Omega}{2\Omega^{\frac{1}{2}}(\Omega-1)^{\frac{3}{2}}} \right] \times$$

$$\left\{ 1 + \frac{2}{\pi} \frac{(\Omega-1)^{\frac{1}{2}}}{2-\Omega} - \frac{2}{\pi} \arctan(\Omega-1)^{\frac{1}{2}} \right\} + O(\epsilon^2) \quad (2.5)$$

This prediction, for $|T/\epsilon|$, agrees very well with the results of Maidanik et al [2]. The transmission, given in Table 2, is substantial only near $\Omega = 1$, dropping to $\epsilon e^{-\pi i/2} / (2^{\frac{1}{2}}\pi) = 0.225 \epsilon e^{-\pi i/2}$ at $\Omega = 2$ and decreasing like

$$\frac{7}{3\pi} \epsilon e^{-\pi i/2} \Omega^{-5/2}$$

as $\Omega \rightarrow \infty$.

TABLE 2 - VALUES OF $|T^{(\text{leaky})}/\epsilon|$ CALCULATED
FROM EQUATION (2.5)

Ω	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$ T^{\text{leaky}}/\epsilon $	13.96	4.44	2.20	1.32	0.87	0.62	0.46	0.35	0.28	0.225

We argue, however, that it must not be concluded that only the leaky wave is significantly excited when $\Omega > 1$. That conclusion might be reached from study of a force-excited panel where the input spectrum is white in the wavenumber domain, and where the lower impedance of the leaky wave (compared with the subsonic wave) ensures that the latter is only weakly excited. If, on the other hand, the input spectrum is not white, but is heavily concentrated in the subsonic wavenumber range $k > k_0$, with hardly any energy in $k < k_0$, then it is possible that the subsonic wave will be favored over the leaky wave. This may be the case with hydrodynamic excitations of the system (turbulent boundary layer pressures for example), though such excitations will generally have less spectral energy around the marginally subsonic wavenumber k_1 than at the characteristic hydrodynamic wavenumber. Consequently it is essential that the various wave modes generated should be separated out the the transmission established separately for each. More will be said about this aspect (and about mode conversion, subsonic to leaky and vice versa, by the rib) in Part II of this paper.

Next the situation near coincidence is analyzed, and an approximation is found which matches smoothly into the small value $T_\Omega = 0(\epsilon)$ of (2.3) for $\Omega < 1$ and into the unit value of T_Ω for $\Omega > 1$. The transmission refers, for $\Omega < 1$ and $\Omega > 1$, to the subsonic free wave. The nonuniformity is characterized by

$$|1-\Omega| = 0(\epsilon^{2/3})$$

and described by approximations in which the magnified frequency variable

$$\lambda = \frac{1-\Omega}{\epsilon^{2/3}} \quad (2.6)$$

is held fixed as $\epsilon \rightarrow 0$. Using the results of Section 5 of [5] one finds

$$T_\lambda = \frac{p_1}{(3p_1 + 2\lambda)} \quad (2.7)$$

plus a remainder term which is $0(\epsilon^{2/3})$ if $\lambda > \lambda_c = (27/4)^{1/3}$ but $0(\epsilon^{1/3})$ if $\lambda < \lambda_c$ (including $\lambda < 0$). Here $p_1(\lambda)$ is the (unique) real positive root of

$$F(q_0) \equiv q_0^3 + \lambda q_0^2 - 1 = 0 \quad (2.8)$$

and is one of the roots given explicitly by

$$\begin{aligned}
& -\frac{\lambda}{3} + \frac{2\lambda}{3} \cos\left\{\frac{1}{3} \arccos\left(\frac{27}{2\lambda^3} - 1\right)\right\} \\
& -\frac{\lambda}{3} + \frac{2\lambda}{3} \cos\left\{\frac{2\pi}{3} - \frac{1}{3} \arccos\left(\frac{27}{2\lambda^3} - 1\right)\right\} \\
& -\frac{\lambda}{3} + \frac{2\lambda}{3} \cos\left\{\frac{4\pi}{3} - \frac{1}{3} \arccos\left(\frac{27}{2\lambda^3} - 1\right)\right\}
\end{aligned} \tag{2.9}$$

Around the critical value λ_c there is a mild nonuniformity at higher order, but this does not affect leading order terms.

Observe that T_λ is real and $O(1)$ when $\lambda = O(1)$. It can be shown (see [5] for details) that as $\lambda \rightarrow +\infty$, expression (2.7) decreases to the value $\epsilon/[2(1-\Omega)^{3/2}]$, matching imperceptibly the asymptotic behavior of T_Ω as Ω increases towards 1 from below. Expression (2.7) also matches smoothly to the unit value of T_Ω for $\Omega > 1$ (indeed a higher-order matching to the two terms quoted in (2.4) was carried out in [5]). To see the general features, note that

$$\frac{\partial}{\partial \lambda} T_\lambda = - \frac{6}{p_1 (3p_1 + 2\lambda)^3}$$

from which it follows (since $3p_1 + 2\lambda \neq 0$) that T_λ increases monotonically from 0 to 1 as λ decreases from $+\infty$ to $-\infty$.

In summary then, the transmission coefficient rises smoothly and monotonically from a value $O(\epsilon)$ to essentially unity, the transition being complete over the range $|1-\Omega| = \epsilon^{2/3}$. Precise values are easily computed from (2.7); in particular we have the interesting result that

$$T_\lambda = \frac{1}{3} + O(\epsilon^{1/3}) \quad (2.10)$$

at $\lambda = 0$ (where the roots of (2.8) are $p_1 = 1$, $p_2 = e^{2\pi i/3}$, $p_3 = e^{4\pi i/3}$), i.e., exactly at $\Omega = 1$.

We turn next to the nonuniformity in (2.3) as $\Omega \rightarrow 0$. It turns out that there is first a nonuniformity when $\Omega = O(\epsilon)$, then a later one when $\Omega = O(\epsilon^2)$. Frequencies $\Omega = O(\epsilon)$ are designated as "intermediate" and a scaling $\Omega = \epsilon\Delta$ is employed together with expansions as $\epsilon \rightarrow 0$ for Δ fixed. The transmission coefficient in this case turns out to be

$$T_\Delta = -\frac{\epsilon^{1/2}}{\Delta^{1/2}(1+\Delta^2)^{1/2}} + \frac{1\epsilon^{1/2}}{\pi\Delta^{1/2}} \left\{ 1 - 2 \ln \left[\frac{2}{(\epsilon\Delta)^{1/2}} \right] \right. \\ \left. + \frac{2}{(1+\Delta^2)^{1/2}} \ln \left(\frac{1 + (1+\Delta^2)^{1/2}}{\Delta} \right) \right\} + \frac{\epsilon^{1/2}}{\Delta^{1/2}} + O(\epsilon) \quad (2.11)$$

This expression matches the formula (2.3) as Δ becomes large and Ω becomes small. The transmission T_Δ is larger, $O(\epsilon^{1/2})$, than T_Ω , but expression (2.11) is still not good down to arbitrarily low frequencies, as it continues to predict infinite transmission as $\Delta \rightarrow 0$. Because of the term involving $\ln \epsilon$, (2.11) is not quite in the form $(\epsilon^{1/2})$ times (a universal function of Δ alone), but it is effectively in that form as the $\ln \epsilon$ term varies very slowly.

Next the "low-frequency" region $\Omega = O(\epsilon^2)$ is examined, taking $\Omega = \epsilon^2 \tilde{\omega}$ with expansions for fixed $\tilde{\omega}$ as $\epsilon \rightarrow 0$. The descriptions are complicated and dealt with in detail in [5, Section 7]. Define

$$G(q_0) \equiv q_0^2(1+q_0) - \frac{1}{\tilde{\omega}} \quad (2.12)$$

and denote the roots of $G(q_0) = 0$ by s_1, s_2, s_3 . In all cases s_1 is used to refer to the unique real positive root ($\tilde{\omega}$ being always positive), while s_2, s_3 are real and negative if $\tilde{\omega} > \tilde{\omega}_c = 27/4$ and form a conjugate pair $-\alpha \pm i\beta$, with $\alpha, \beta > 0$, when $\tilde{\omega} < \tilde{\omega}_c$. Define also

$$S = \sum_{n=1}^3 \frac{\ln|s_n|}{s_n(3s_n+2)} \quad (2.13)$$

and $\Theta = \arctan(\beta/\alpha)$, $0 < \Theta < \pi/2$. Then for $\tilde{\omega} > \tilde{\omega}_c$ one has the following expression for the transmission coefficient

$$T_{\tilde{\omega}} = \left[\frac{S}{S + \frac{i\pi}{s_1(3s_1+2)}} \right] + O(\epsilon^2) \quad (2.15)$$

while when $0 < \tilde{\omega} < \tilde{\omega}_c$

$$T_{\tilde{\omega}} = \frac{\left[S + \Theta \tilde{\omega}^{\frac{1}{2}} \frac{(3s_1+1)}{(3s_1-1)^{\frac{1}{2}}(3s_1+2)} \right]}{\left[S + \Theta \tilde{\omega}^{\frac{1}{2}} \frac{(3s_1+1)}{(3s_1-1)^{\frac{1}{2}}(3s_1+2)} + \frac{i\pi}{s_1(3s_1+2)} \right]} + o(1) \quad (2.16)$$

(it being difficult to improve upon the $o(1)$ estimate of the error when $\tilde{\omega} < \tilde{\omega}_c$). After extensive manipulation it is possible to rewrite both (2.15) and (2.16) in terms solely of the root s_1 (which determines the free subsonic wavenumber), using the expressions

$$S = \frac{\left[\ln \frac{s_1}{s_1+1} \right]}{2s_1(3s_1+2)} \quad (2.17)$$

for $\tilde{\omega} > \tilde{\omega}_c$, and

$$S = \frac{\left[\ln \frac{s_1}{s_1+1} \right]}{2s_1(3s_1+2)} + \left[\frac{1-3s_1}{1+s_1} \right]^{\frac{1}{2}} \left[\frac{1+3s_1}{1-3s_1} \right] \times$$

$$\frac{\ln \left[\frac{(1+s_1)^{\frac{1}{2}} + (1-3s_1)^{\frac{1}{2}}}{(1+s_1)^{\frac{1}{2}} - (1-3s_1)^{\frac{1}{2}}} \right]}{2s_1(3s_1+2)}, \quad (2.18)$$

$$\Theta = \arctan \left[\frac{3s_1-1}{s_1+1} \right]^{\frac{1}{2}} \quad (2.19)$$

for $\tilde{\omega} < \tilde{\omega}_c$, noting that $3s_1 - 1 \leq 0$ according as $\tilde{\omega} \gtrless \tilde{\omega}_c$ so that all radicals in these expressions are real and positive. These expressions look complicated, but can be easily evaluated using the Table of values of s_1 given in reference [5]. They show that in the frequency range $\Omega = O(\epsilon^2)$, an $O(1)$ fraction of the incident energy is transmitted across the rib, and fluid loading effects have again managed to "mend" the

rib discontinuity even though the fluid loading parameter ϵ (which actually measures fluid mass divided by surface mass at the coincidence frequency) is small.

Such examination of higher terms in the "low-frequency expansion" as it is possible to carry out indicates that there are no further leading order nonuniformities (in the transmission coefficient, at any rate), and that (2.16) should be good down to zero frequency. If the limit $\tilde{\omega} \rightarrow 0$ is taken, it is found, on reduction, that

$$T_{\tilde{\omega}} \rightarrow \frac{1}{2} e^{-\pi i/3} \quad \text{as } \Omega \rightarrow 0 \quad (2.20)$$

This result is in fact true, for sufficiently low frequencies, whatever the value of the parameter ϵ . It shows just how substantial the transmission becomes in the "heavy fluid loading" limit.

There is a nonuniformity in these results at higher order in a narrow region around the value

$$\tilde{\omega} = \tilde{\omega}_c = 27/4 \quad (2.21)$$

where one defines

$$\tilde{\omega} = \frac{27}{4}(1 + \epsilon^2 \mathcal{J}) \quad (2.22)$$

and considers $\mathcal{J} = O(1)$ as $\epsilon \rightarrow 0$. The nonuniformity does not affect the leading order terms, however, and it is found that

$$T_g = \frac{1 - \frac{1}{3} \ln 4}{1 - \frac{1}{3} \ln 4 + \frac{2i\pi}{3}} + O(\epsilon) \quad (2.23)$$

with $|T_g| = 0.25 + O(\epsilon)$. This higher order nonuniformity manifests itself in the numerical computations of [2], where in Figure 2 one notices a narrow plateau, centered on $\Omega = (0.26)^2 = 0.068$ which, for $\epsilon = 0.1$ (taken in [2]) is almost identical with our $\tilde{\omega} = \tilde{\omega}_c$, and at which $|T|$ has a local maximum of 0.27 in good agreement with the prediction above of 0.25. As remarked in [5], one might have suspected the curious little plateau of arising from some numerical inaccuracies, but the analysis here shows that it is indeed genuine.

The overall pattern of free subsonic wave energy transmission is summarized schematically in Figure 1, in which is also included, for $\Omega > 1$, the transmission coefficient for leaky waves. The various approximations were shown in [5] to overlap in the manner indicated in Figure 1. This Figure largely speaks for itself, and the only further comment which needs to be made is that it is possible to identify the physical mechanisms which dominate the various approximations. Inasmuch as it is possible to express the admittances entirely in terms of a wavenumber k_1 of a free subsonic surface wave, we need only quote the mechanisms which dominate in determining that wavenumber (in contrast to [5], where a slightly more elaborate description was given). Our model has retained the following five possible physical attributes: (I) membrane stiffness, (II) membrane mass, (III) fluid pressure forces, (IV) fluid compressibility, and (V) fluid mass. For $0 < \Omega < 1$ the dominant processes are (I)

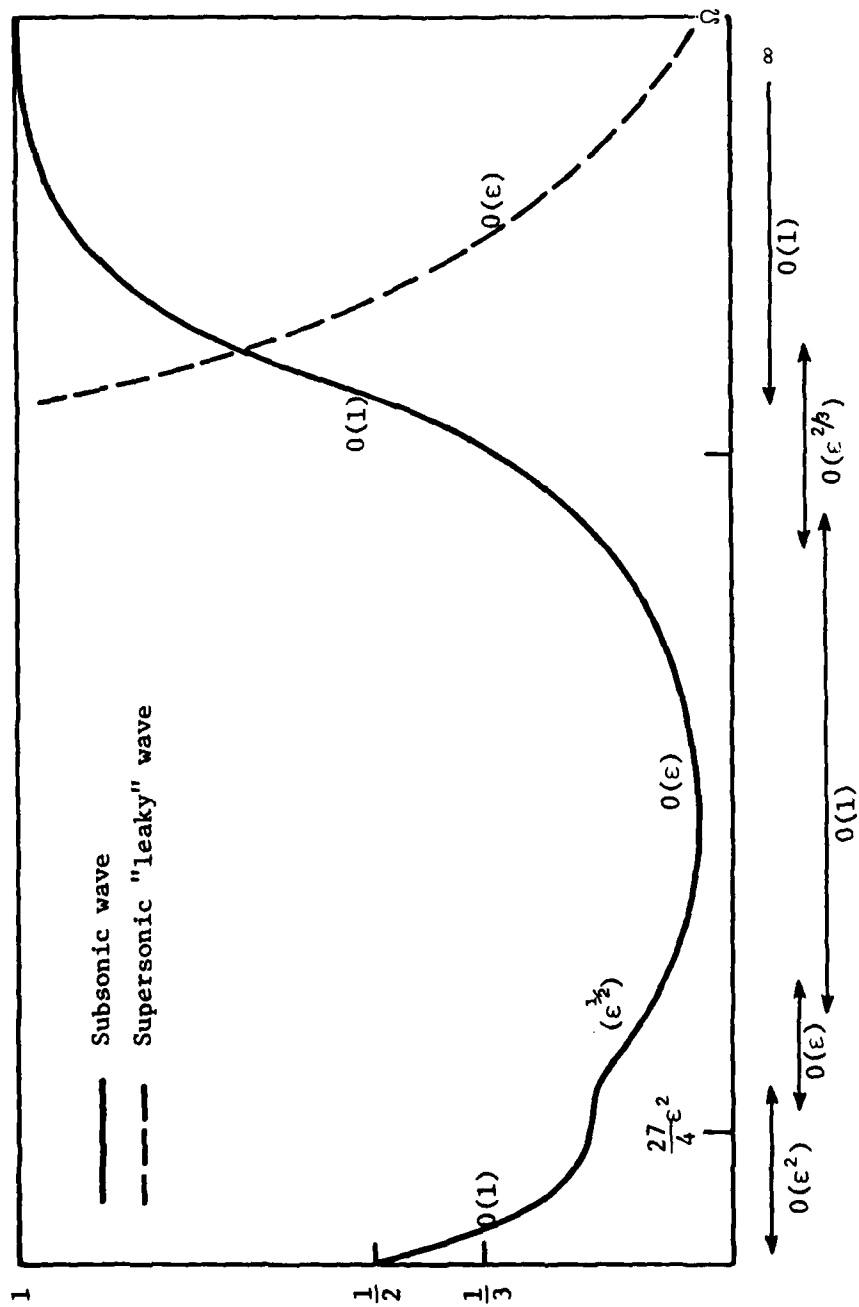


Figure 1 - Schematic Presentation of Transmission Coefficient Magnitude (for $\epsilon \ll 1$) as Function of Frequency.

and (II); for $\Omega > 1$ they are (III) and (IV) for the subsonic wave, but (I) and (II) for the leaky wave; for $|1-\Omega| = O(\epsilon^{2/3})$ all five processes are significant; for $\Omega = O(\epsilon)$ only (I) and (II) are important; for $\Omega = O(\epsilon^2)$ only (IV), fluid compressibility, can be neglected; and when $\Omega \rightarrow 0$ the mechanisms (I), (III), and (V) determine the heavy loading limit.

This completes our study of normal incidence energy transmission across a single rib; oblique incidence will be discussed in Section 5. We turn now to acoustic aspects of the action of the rib.

3. THE ACOUSTIC FIELD SCATTERED BY A RIB

In this section we deal with the acoustic fields generated when a free subsonic wave is incident normally upon the rib, and when a plane acoustic wave is incident upon the panel with the single rib, the latter being determined by the so-called "non-specular reflection coefficient." Both acoustic fields are readily derived from the field generated by a single prescribed line force F acting on the panel, that field being given by the Fourier integral

$$p(x,y) = - \frac{\rho_0 \omega^2 F}{2\pi T} \int_{-\infty}^{+\infty} \frac{\exp\{ik|x| - (k^2 - k_0^2)^{1/2} y\}}{(k^2 - k_m^2)(k^2 - k_0^2)^{1/2} - \mu k_m^2} dk \quad (3.1)$$

The notation is standard except for μ , equal to ρ_0/m where m is the specific mass of the panel. Also one has $k_m^2 = m\omega^2/T$ with T the membrane tension, and $k_0 = \omega/c_0$. Evaluation of this integral by steepest descents or stationary phase gives an acoustic farfield

$$p(r, \theta) \sim - \frac{\rho_o \omega^2 F}{2\pi T} \frac{(2k_o \pi/r)^{\frac{1}{2}} e^{ik_o r - \pi i/4} \sin \theta}{[(k_o^2 \cos^2 \theta - k_m^2)(-ik_o \sin \theta) - \mu k_m^2]} \quad (3.2)$$

with $|x| = r \cos \theta$, $y = r \sin \theta$, $0 < \theta < \pi/2$. This estimate is valid for all θ if r is sufficiently large. If $\epsilon \ll 1$ and $0 < \Omega < 1$ it is probably enough to take $k_o r \gg 1$, but when $\Omega > 1$ the estimate holds away from the Mach directions $\theta = \arccos \Omega^{-\frac{1}{2}}$ when $k_o r \gg 1$, and close to those directions only when $k_o r$ is very large, $k_o r \gg \epsilon^{-2}$. The reader is referred to [7] for further details of the Mach wave field associated with leaky waves in the surface, especially for a description holding when $k_o r$ is not as large as ϵ^{-2} .

Here Eq. (3.2) will be accepted as describing "the distant acoustic field" in an appropriate, known sense. Then

$$|p|^2 = \frac{\mu^2 F^2}{2\pi k_o r} H(\Omega, \epsilon, \theta) \quad (3.3)$$

with

$$H(\Omega, \epsilon, \theta) = \frac{\sin^2 \theta}{[(\Omega \cos^2 \theta - 1)^2 \sin^2 \theta + \frac{\epsilon^2}{\Omega}]} \quad (3.4)$$

and we now state, very briefly, leading order approximations to the directivity function H .

(1) $\Omega > 1$. Define a Mach angle $\theta_M = \arccos \Omega^{-\frac{1}{2}}$ and write $\theta = \theta_M + \epsilon \chi$ when θ is close to θ_M . If $|\theta - \theta_M|$ is not as small as $O(\epsilon)$

$$H \sim \frac{1}{(\Omega \cos^2 \theta - 1)^2} \quad (3.5)$$

except when θ is small, $\theta = 0(\epsilon) = \epsilon\psi$ say, and for this case

$$H \sim \frac{\psi^2}{(\Omega-1)^2 \psi^2 + \frac{1}{\Omega}} \quad (3.6)$$

This vanishes for $\psi \rightarrow 0$, a manifestation of the well-known "Lloyd's mirror" effect for grazing incidence acoustic waves.

If $|\theta - \theta_M| = 0(\epsilon)$, so that $\chi = 0(1)$, then

$$H \sim \frac{\sin^2 \theta_M}{\epsilon^2 [4\chi^2 \Omega^2 \cos^2 \theta_M \sin^4 \theta_M + \frac{1}{\Omega}]} \quad (3.7)$$

whose $0(\epsilon^{-2})$ largeness demonstrates the intense field at very great distances ($k_0 r \gg \epsilon^{-2}$) within $0(\epsilon)$ of the Mach directions as compared with the $0(1)$ fields (3.5) and (3.6) elsewhere.

(2) $0 < \Omega < 1$. Here the fluid loading term ϵ^2/Ω must be retained only for $\theta = 0(\epsilon)$, where it leads again to the form (3.6). This low angle nonuniformity arises in all cases, and will receive no further comment. For θ larger than $0(\epsilon)$,

$$H \sim \frac{1}{(1 - \Omega \cos^2 \theta)^2} \quad (3.8)$$

which has only a weak dependence upon θ and indicates fairly uniform directivity.

(3) $|1-\Omega| = O(\epsilon^{2/3})$. No special phenomena arise in this case. The Mach angle θ_M is $O(\epsilon^{1/3})$, whereas the Lloyd's mirror effect operates only for small angles $\theta = O(\epsilon)$, so that the Mach wave field continues to be as described in (1) above, the field for $\Omega < 1$ as in (2) above.

(4) $\Omega = O(\epsilon)$. Here, writing $\Omega = \epsilon\Delta$ and letting $\epsilon \rightarrow 0$ one finds

$$H \sim 1 \quad (3.9)$$

and the field is isotropic (except for $\theta = O(\epsilon)$).

(5) $\Omega = O(\epsilon^2)$. Put $\Omega = \epsilon^2\tilde{\omega}$ and let $\epsilon \rightarrow 0$. Then for all angles, including $\theta = O(\epsilon)$,

$$H \sim \frac{\sin^2\theta}{\sin^2\theta + \frac{1}{\tilde{\omega}}} \quad (3.10)$$

which indicates a significant effect of fluid loading on the field at all angles, that effect leading to a dipole type of behavior,

$$H \sim \tilde{\omega} \sin^2\theta \quad (3.11)$$

when $\tilde{\omega} \gtrsim 1$.

Most of the results given here are, of course, well-known, though it may be helpful to have them set out as above, as this makes it clear just when the different approximations hold.

To relate these results to the field scattered when a subsonic surface wave is incident upon a rib, all one has to do is to substitute the expression

$$F = \frac{V_o}{A_r - A_o} \quad (3.12)$$

given in Section 2, into (3.3). Here V_o is the amplitude of the incident wave, A_r the admittance of the rib, and A_o the drive point admittance of the fluid-loaded surface. The dependence of A_o upon Ω and ϵ has been thoroughly explored in [5], while A_r is a specified function of Ω alone. The substitution of (3.12) into (3.3) therefore changes only the level of the sound pressure in a fully known way, and does not change the directivity, which is completely defined by $H(\Omega, \epsilon, \theta)$.

As for the non-specular reflection problem, suppose that a plane acoustic wave with pressure

$$p_o \exp(ik_o x \cos\theta_o - ik_o y \sin\theta_o)$$

is incident at an angle θ_o to the surface. Then it is easy to show (a general formulation for the case of many ribs is given in [6]) that the non-specular field scattered by the rib is again given by (3.1) with

$$F = - \frac{ik_o \sin\theta_o (1-R_*)}{(A_r - A_o)} \left(\frac{p_o}{\rho_o i\omega} \right) \quad (3.13)$$

where

$$R_* = \left(\frac{i\Omega^{\frac{1}{2}} \sin\theta_o (\Omega \cos^2 \theta_o - 1) - \epsilon}{i\Omega^{\frac{1}{2}} \sin\theta_o (\Omega \cos^2 \theta_o - 1) + \epsilon} \right) \quad (3.14)$$

is the plane wave reflection coefficient. This value for F is of the same form as (3.12), with V_o in that expression replaced by the velocity

$$\left(\frac{P_o}{\rho_o i \omega} \right) (-i k_o \sin \theta_o)$$

which would be generated at $x = 0$ by incident and reflected acoustic waves if the rib were not present. The directivity function, of incidence angle θ_o and observation angle θ , arising from (3.2) with (3.12) and (3.14), is called the non-specular reflection coefficient. Its dependence upon θ has already been dealt with. The dependence of the squared magnitude of this coefficient upon incidence angle is as

$$\frac{\epsilon^2 \sin^2 \theta_o}{\epsilon^2 + \Omega \sin^2 \theta_o (\Omega \cos^2 \theta_o - 1)^2}$$

which is, as expected from Reciprocity requirements, identical with the θ -dependence. The behavior of the non-specular reflection coefficient is therefore already described by the remarks above relating to $H(\Omega, \epsilon, \theta)$.

4. EFFECTS OF FINITE RIB IMPEDANCE

In the free wave energy transmission problem considered in Section 2, there will be substantial direct "mechanical" transmission across a rib unless the rib has very high mechanical impedance. We have seen that in certain frequency regimes there is also substantial transmission, by virtue of fluid loading effects, even when the rib has infinite impedance. The question to be addressed here is the following: "In those frequency regimes, is it possible to reduce the transmission by balancing fluid-loading effects against finite impedance effects?"

In terms of the impedance $Z_r = A_r^{-1}$ of the rib, the normal incidence transmission coefficient is

$$T = \frac{1 + Z_r (A_\infty - A_0)}{1 - Z_r A_0} \quad (4.1)$$

in which we write

$$Z_r = -R - iX \quad (4.2)$$

for the resistive and reactive components, so that for a simple damped mass-spring system

$$X = \frac{1}{\omega} (K - M\omega^2) \quad (4.3)$$

and K, R, M are all positive constants.

Near coincidence, i.e., for $|1 - \Omega| = O(\epsilon^{2/3})$, there is significant transmission, and it is found from the admittance approximations given in [5] that here

$$T = \frac{1 + \left(\frac{k_m}{m\omega} \right) \frac{1}{2} \left(\frac{p_1}{3p_1 + 2\lambda} \right) (R + iX)}{1 + \left(\frac{k_m}{m\omega} \right) \frac{1}{2} (R + iX)} \quad (4.4)$$

The quantity $p_1/(3p_1 + 2\lambda)$ is the transmission coefficient for infinite rib impedance, and increases from very small positive values to unity as the frequency increases through a range $O(\epsilon^{2/3})$ around the coincidence frequency. It is clear that there can be no choice of R, X which will lead to very substantial reductions in the value of $|T|$ in the range where $p_1/(3p_1 + 2\lambda)$

is close to unity. That is really not surprising, for around the coincidence frequency the surface wave is essentially a plane acoustic wave in the fluid, with very high impedance $\rho_0 \omega / (k^2 - k_0^2)^{1/2}$ in directions normal to the surface. The rib will therefore not be able to significantly affect the propagation of that wave, whatever the rib impedance.

Consider, on the other hand, very low frequencies, where fluid loading effects are dominant and where the infinite-impedance transmission is given by (2.20). Here one finds

$$T = \frac{1 - \frac{1}{3\sqrt{3}} \left(\frac{k_m}{m\omega} \right) \tilde{\omega}^{1/6} (R+iX)}{1 + \frac{1}{3} \left(1 - \frac{1}{\sqrt{3}} \right) \left(\frac{k_m}{m\omega} \right) \tilde{\omega}^{1/6} (R+iX)} \quad (4.5)$$

from which it can be seen that the choices

$$R = 0, \quad 1 + \left(\frac{k_m}{m\omega} \right) X \frac{\tilde{\omega}^{1/6}}{3\sqrt{3}} = 0 \quad (4.6)$$

minimize T , and in fact make $T = 0$. The negative value of X required for (4.6) implies that the rib is driven below its resonance frequency; the balance expressed by (4.6) is then essentially between the stiffness-like behavior of the imaginary part of the surface drive point impedance and the mass-controlled imaginary part of the rib impedance. (Attention was drawn in [5] to the surprising fact that $\text{Im } A_0 < 0$ for the fluid-loaded membrane throughout the whole frequency range, implying that fluid loading acts as an equivalent stiffness.) Equation (4.6) (or its analog for a more realistic type of panel) might only be capable of satisfaction over a very narrow frequency band, if at all; there is, nonetheless, a

clear possibility of reducing the low-frequency free wave transmission by using a mass-controlled rib impedance to at least partially cancel the fluid-loading induced drive point stiffness.

5. FREE WAVE ENERGY TRANSMISSION; OBLIQUE INCIDENCE

The rib lies along the z -axis, and we now assume that the whole wave field has the dependence $\exp(i\ell z)$, so that, for example, ℓ is the z -wavenumber of a free surface wave obliquely incident on the rib.

It is then easy to see that any problem with this z -dependence can be reduced to a purely two-dimensional one in the (x, y) plane, provided that in the two-dimensional problem we make the replacements

$$\left. \begin{aligned} k_m &\rightarrow K_m = (k_m^2 - \ell^2)^{\frac{1}{2}} \\ k_o &\rightarrow K_o = (k_o^2 - \ell^2)^{\frac{1}{2}} \\ \mu &\rightarrow \nu = \mu k_m^2 / K_m^2 \end{aligned} \right\} \quad (5.1)$$

and

Since, in the original integral defining the surface response (Eq. 2.10 of [5]), the points k_o and k_m must lie above the path of integration, it follows that we must take $K_m = +i(\ell^2 - k_m^2)^{\frac{1}{2}}$, $K_o = +i(\ell^2 - k_o^2)^{\frac{1}{2}}$ when K_m and K_o are not real. For normalized functions which depend only on the two parameters Ω and ϵ (equivalent to the ratios of the three wavenumbers in (5.1)) the correspondence (5.1) may be expressed as

$$\left. \begin{aligned} \Omega &\rightarrow \Omega_* = \left(\frac{\Omega - L^2}{1 - L^2} \right) \\ \epsilon &\rightarrow \epsilon_* + \epsilon \left(\frac{\Omega_*}{\Omega} \right)^{\frac{1}{2}} \frac{1}{(1 - L^2)^{3/2}} \end{aligned} \right\} \quad (5.2)$$

where

$$l = k_m L$$

Consider first the wavenumber in the x-direction of a free surface wave at frequency ω ; for that wavenumber to be real, l must not exceed the total wavenumber magnitude of a free wave. In reference [5] approximations were given for the total wavenumber, so that one has the following values of the maximum permissible z-wavenumber:

$$\underline{\Omega > 1}: \quad l_{\max} = k_o \left\{ 1 + \frac{\epsilon^2}{2\Omega^2 (\Omega - 1)^2} + O(\epsilon^4) \right\} \quad (5.3a)$$

$$\underline{0 < \Omega < 1}: \quad l_{\max} = k_m \left\{ 1 + \frac{\epsilon}{2\Omega^{\frac{1}{2}} (1 - \Omega)^{\frac{1}{2}}} + O(\epsilon^2) \right\} \quad (5.3b)$$

$$\underline{\Delta = \frac{\Omega}{\epsilon} = O(1)}: \quad l_{\max} = k_m \left\{ 1 + \frac{\epsilon^{\frac{1}{2}}}{2\Delta^{\frac{1}{2}}} + O(\epsilon) \right\} \quad (5.3c)$$

$$\underline{\tilde{\omega} = \frac{\Omega}{\epsilon^2} = O(1)}: \quad l_{\max} = k_m / \tilde{\omega}^{\frac{1}{2}} s_1 \quad (5.3d)$$

(where s_1 is the positive root of $s^3 + s^2 - \frac{1}{\tilde{\omega}} = 0$)

$$\underline{\tilde{\omega} \rightarrow 0}: \quad l_{\max} \sim k_m / \tilde{\omega}^{1/6} \rightarrow \infty \quad (5.3e)$$

$$\lambda = \frac{1-\Omega}{\epsilon^{2/3}} = 0(1): \quad \ell_{\max} = k_m \{1 + \frac{1}{2}\epsilon^{2/3} p_1 + 0(\epsilon^{4/3})\} \quad (5.3f)$$

(where p_1 is the positive root of $p^3 + \lambda p^2 - 1 = 0$).

One sees then from (5.3) that for frequencies $0(\epsilon^2)$ there is a change, by an $0(1)$ factor, in the maximum permissible ℓ , from its value k_m in the absence of fluid loading, and that as $\Omega \rightarrow 0$ this factor tends (slowly) to infinity. In this "heavy fluid loading" limit, all waves can propagate in the x-direction, no matter how large their z-wavenumber. In all other frequency ranges below coincidence the fluid loading correction to the vacuum value k_m is small, and corresponds to an increase associated with the addition of fluid mass to surface mass. For $\Omega > 1$, a wave propagates in the x-direction with no attenuation if (5.3a) is satisfied, and the wave is subsonic. If, however, the wave is an oblique subsonic leaky wave, its total wavenumber is

$$k_m \left\{ 1 + \frac{1\epsilon}{2\Omega^{1/2}(\Omega-1)^{1/2}} + 0(\epsilon^2) \right\},$$

and therefore it will propagate in the x-direction with only slow attenuation, $0(\exp(-\epsilon x))$, if $\ell < k_m$, but with strong attenuation if $\ell > k_m$. Thus we can take

$$\underline{\Omega > 1}: \quad \ell_{\max} = k_m \quad (5.3g)$$

as the condition for a leaky wave to "propagate" in the x-direction.

Results for the transmission coefficient across a rib of infinite impedance were presented in Section 2, as functions of ϵ and Ω . It is straightforward then to use (5.2) to immediately produce results for oblique incidence, though the results are algebraically complicated. Suppose first that $\Omega > 1$; then if $L^2 < 1$, $\Omega_* > 1$ and therefore $T = 1 + O(\epsilon^2)$ from (2.4), while if $L^2 > 1$, $\Omega_* < 1$ and therefore $T = O(\epsilon_*)$ from (2.3). Since $\epsilon_* = O(\epsilon)$ unless L^2 is close to Ω or to 1, we can say that when $\Omega > 1$ an oblique wave with $L^2 < 1$ is essentially totally transmitted (as for normal incidence), but has a transmission coefficient of only $O(\epsilon)$ if $L^2 > 1$.

Suppose next that $0 < \Omega < 1$, with Ω not close to 0 or 1. When L is close to 1 a separate examination is needed anyway, so that we can say, from (5.3b) that $L^2 < 1$. Then $\Omega_* < 1$, and Eq. (2.3) gives the transmission coefficient except when Ω_* is small, that is, when L^2 is close to Ω . We have made a detailed analysis of the latter possibility, using a scaling $L^2 = \Omega - \rho\epsilon^2$ and keeping ρ fixed as $\epsilon \rightarrow 0$. By direct calculation of the case $L^2 = \Omega$ we confirm that expansions in the ρ -variable are in fact valid down to $\rho = 0$. Throughout the region $\rho = O(1)$ it is found that the admittances are given by

$$A_o = \frac{\omega}{Tk_m} \frac{1}{2D^{\frac{1}{2}}} + O(\epsilon) \quad (5.4a)$$

$$A_\infty = \frac{\omega}{Tk_m} \frac{1}{2D^{\frac{1}{2}}} + O(\epsilon) \quad (5.4b)$$

and hence that $T = O(\epsilon)$. Specifically, when $L^2 = \Omega$ we find

$$T = \frac{21}{\pi} \left(\frac{\nu}{K_m} \right) \left[\frac{1}{4} + \ln \left(\frac{\nu}{K_m} \right) \right] + O(\nu^2/K_m^2) \quad (5.4c)$$

with $\nu/K_m = (\epsilon/\Omega^{\frac{1}{2}})(1-L^2)^{-3/2}$.

The prediction $T = O(\epsilon)$ comes also from use of (2.3) away from $L^2 = \Omega$, so that we conclude that, except perhaps for $L^2 \approx 1$, but including the case $L^2 \approx \Omega$, the transmission coefficient is small, of order ϵ , for both normally and obliquely incident waves with frequencies satisfying $0 < \Omega < 1$.

As Ω decreases further, the transmission coefficient increases, becoming $O(\epsilon^{\frac{1}{2}})$ when $\Omega = O(\epsilon)$ and $O(1)$ when $\Omega = O(\epsilon^2)$, just as for normal incidence. The functional forms, of course, vary with L , but this does not change the overall picture, and in fact as $\Omega \rightarrow 0$ the dependence on L drops out, and the transmission coefficient is given by (2.20) for all waves.

The case $L \approx 1$ remains to be dealt with. It turns out that two "inner" expansions are needed to cover the nonuniformity here, reflecting the fact that the basic approximation (2.3) is known to be nonuniform when $\epsilon \sim \Omega$ and when $\epsilon^2 \sim \Omega$. If $\epsilon_* \sim \Omega_*$ as $L^2 \rightarrow 1$ one finds that $L^2 - 1 = O(\epsilon)$, while if $\epsilon_*^2 \sim \Omega_*$ as $L^2 \rightarrow 1$ one must have $L^2 - 1 = O(\epsilon^{2/3})$. One therefore defines $L^2 = 1 - \gamma\epsilon^{2/3}$, say, and constructs approximations to the admittances with γ fixed as $\epsilon \rightarrow 0$. These approximations cannot be gotten from approximations like (2.3); indeed the point of introducing the γ -variable is to remedy the deficiencies of (2.3) when γ is $O(1)$, so that the new approximations have to be derived from exact results given in [5], and then matched to the limiting behavior of (2.3). The process is straightforward, but lengthy, and leads to admittances

$$A_0 = \frac{\omega}{Tk_m} \left\{ \frac{1}{2\gamma^{\frac{1}{2}} \epsilon^{\frac{1}{3}}} - \frac{1}{4\gamma^{\frac{3}{2}} \Omega^{\frac{1}{2}} D^{\frac{1}{2}}} + O(\epsilon^{\frac{1}{3}}) \right\} \quad (5.5)$$

$$A = \frac{2\omega\mu}{Tk_m^2} \frac{\Omega^{\frac{1}{2}}}{4\gamma^{\frac{1}{2}} \epsilon^{\frac{1}{3}}} \left\{ 1 - \frac{\epsilon^{\frac{1}{3}}}{2\gamma\Omega^{\frac{1}{2}} D^{\frac{1}{2}}} + O(\epsilon^{\frac{2}{3}}) \right\} \quad (5.6)$$

from which we find

$$T = 1 - \frac{A_\infty}{A_0} = O(\epsilon^{\frac{2}{3}}) \quad (5.7)$$

We have not carried the calculation far enough to see if, in fact, the coefficient of the $O(\epsilon^{\frac{2}{3}})$ term in T is zero (which would then lead to $T = O(\epsilon)$), but at any rate it is clear that the transmission is still small when $T = O(1)$.

The approximations (5.5) and (5.6) do not apply right down to $L^2 = 1$ (i.e., $\gamma = 0$), as the ratio of consecutive terms is $O(\epsilon^{\frac{1}{3}}/\gamma)$ and not uniformly small. To remedy this one writes $L^2 = 1 - \kappa\epsilon$, constructs approximations for $\kappa = O(1)$ and matches them to the asymptotic behavior of (5.5) and (5.6) as $\gamma \rightarrow 0$. The results are

$$A_0 = \left(\frac{\omega}{Tk_m} \frac{1}{2\epsilon^{\frac{1}{2}}} \left(\kappa + \frac{1}{\Omega^{\frac{1}{2}} D^{\frac{1}{2}}} \right)^{-\frac{1}{2}} + O(\epsilon^{\frac{1}{2}}) \right) \quad (5.8)$$

$$A_\infty = \left(\frac{2\omega\mu}{Tk_m^2} \frac{\Omega^{\frac{1}{2}}}{4\epsilon^{\frac{1}{2}}} \left(\kappa + \frac{1}{\Omega^{\frac{1}{2}} D^{\frac{1}{2}}} \right)^{-\frac{1}{2}} + O(\epsilon^{-\frac{1}{2}}) \right), \quad (5.9)$$

$$T = O(\epsilon)$$

and these results appear to apply right down to $\kappa = 0$, so that no further

"inner" expansions are needed. Although the dependence of T upon κ has not been obtained, even to leading order, it was not felt to be worthwhile to pursue this matter further. The essential point is that there are nonuniformities around $L^2 = \Omega$ and $L^2 = 1$ (reflected, for example, in the much larger values of the admittances as $L^2 \rightarrow 1$), but these nonuniformities affect the drive admittance A_0 and the transfer admittance A_∞ in the same way, and still leave a very small value of the transmission coefficient (for a rib of infinite mechanical impedance) both around $L^2 = \Omega$ and around $L^2 = 1$.

If the rib impedance is not infinite there are many possibilities, but in that case there will be a substantial mechanical transmission of energy, largely independent of fluid loading effects. What has been shown here is that, where fluid loading is the only mechanism for energy transfer, no dramatic changes are associated with oblique incidence; in particular, waves with z -wavenumber close to either k_0 or k_m are not transmitted with significantly different efficiency from normally incident waves.

In the case of finite rib impedance, this will normally depend on the z -wavenumber and on the frequency separately if the rib is, for example, of the string or beam type with nonlocal reaction. Thus in (4.3) the stiffness K could be written as

$$M\omega_0^2(\ell)$$

where the resonance frequency ω_0 is a function of ℓ . For certain ranges of driving frequency ω , the condition $\omega_0 \approx \omega$ may be met for a certain

small range of ℓ , and in that case, provided the resistive part of the rib admittance is then small compared with A_∞ and A_0 , it follows from (4.1) that

$$T \approx 1 \quad (5.10)$$

This almost perfect transmission is the result of mechanical coupling across the rib, and one may ask whether fluid loading effects can significantly change either the magnitude of the transmission, or the frequency bandwidth for which (5.10) holds. To examine this, ignore the resistive term in (4.2), and use the normalized admittances B_0 , B_∞ defined in [5]. Then

$$T = \frac{1 + \left(\frac{M\omega^2}{Tk_m} \right) \left(1 - \frac{\omega_0^2}{\omega^2} \right) B_0 T_\infty}{1 + \left(\frac{M\omega^2}{TK_m} \right) \left(1 - \frac{\omega_0^2}{\omega^2} \right) B_0} \quad (5.11)$$

where T_∞ is the transmission coefficient for infinite rib impedance. Now generally T_∞ is small, while B_0 is then close to its value ($\frac{1}{2}$) in the absence of fluid loading, and then it follows that fluid loading effects on both the magnitude and bandwidth of the "resonant" transmission are negligible. The same applies at frequencies around coincidence, where T_∞ is close to unity; in that case we have $T \approx 1$ regardless of whether the resonance condition $\omega = \omega_0$ is met in this frequency range or not. If we take the very low frequency limit $\omega \rightarrow 0$ we find that B_0 vanishes like $\omega^{1/6}$, while we assume that k_m varies like $\omega^{1/2}$, as it would for a thin plate. Then the second term dominates in both numerator and denominator of

(5.11), and so

$$T \sim T_{\infty}$$

as $\omega \rightarrow 0$. Thus for low frequencies, fluid loading effects (represented by the variation of B_0 with ω) again substantially "mend" the impedance discontinuity, and except for resonant rib conditions, the low-frequency transmission is essentially identical with that for normal incidence and infinite rib impedance. At the resonant rib condition it appears that $T \approx 1$ because of mechanical coupling, and fluid loading effects are not able to significantly offset that coupling.

6. CONCLUSIONS

It is hoped that this analytical study of a very simple model configuration will contribute to understanding of fluid loading effects, both by providing insight into the physical processes, and also by complementing numerical studies (such as reported in [1] and [2]) which tend to run into difficulties—even in simple configurations—precisely where fluid loading effects are most pronounced and most interesting. Highly accurate predictions for specific cases have not been given, though the analytical results given here have been put in a form where all the sensitive behavior has been explicitly extracted, leaving simple functions which can indeed be readily computed in any specific case. Purely numerical computations of Fourier integrals for fluid-loaded plate dynamics can (indeed, are bound to) find great difficulty in resolving the balance between structural and fluid coupling; as remarked above, this happened in the work reported in [1] and [2], both at low

frequencies and close to the coincidence condition. The perturbation method approach shows why such difficulties are likely to arise, shows how a balance is achieved between various competing physical mechanisms in different frequency ranges, and provides (at the least) scalings and estimates of effects which should be useful in computational studies of more realistic configurations.

In Part II of this work we intend to pursue the application of these methods to structures with two or more inhomogeneities.

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